2018 DSA Mid-term Programming Exam

Things to remember before you get started...

- As usual, your program should take the input from the standard input and send the output to the standard output. That is, you should be able to redirect the input/output files like this:

  ./myProgram < input.txt > output.txt
- Since "cin" and "cout" are slower for input/output, you should try to use "scanf" or "printf" as much as possible to avoid TLE.
- Time and memory limits for each problem are listed at the judge system.

And here goes the exam:

1. (30 pts) **Removing same-value neighboring pairs from a vector**: Given a vector of n integers $\{a_1, a_2, \ldots, a_n\}$, we want to remove same-value neighboring pairs by performing the following operations:
   a. Scan the vector from left to right until we locate two same-value neighboring integers.
   b. Remove these two integers from the vector.
   c. Repeat the above two steps until there are no same-value neighboring elements.

   Please output the final vector (from left to right).

   More details:

   - Input format:
     - Line 1 is an integer n, the length of the vector.
     - Line 2 contains n integers of the vector's elements.
   - Output format: The output is a single line containing the final vector, with neighboring elements separated by a single space. In particular, if the result is an empty vector, please print "Meow" (without double quotes) instead.
   - Ranges of variables:
     - $1 \leq n \leq 10^6$
     - $1 \leq a_i \leq 10^6, \forall i.$
   - Example test cases:
     1. Case 1:
        - Sample input:

          ```
          2
          1 1
          ```

        - Sample output:

          ```
          Meow
          ```

     2. Case 2:
        - Sample input:

          ```
          4
          1 1 2 3
          ```

        - Sample output:

          ```
          2 3
          ```

- Subtask descriptions:
  - $n = 1$ ==> 2 pts
  - $n = 2$ ==> 2 pts
  - $n = 3$ ==> 4 pts
  - $a_1 = a_2 = \cdots = a_n$ ==> 2 pts
  - $1 \leq n \leq 10^3$ ==> 10 pts
  - $1 \leq n \leq 10^6$ ==> 10 pts

2. (40 pts) **Median of pairs from a vector**: Given a vector of n integers $\mathbf{a} = \{a_1, a_2, \ldots, a_n\}$, we can form a pair sum vector $\mathbf{b}$ of m $(= n(n-1)/2)$ elements by selecting any 2 elements from vector $\mathbf{a}$ and use their sums as elements. In other words, the elements of $\mathbf{b}$ can be described by the multi-set (which is a set that allows multiple instances of same-value elements):

$$\{a_i + a_j | 1 \leq i < j \leq n\}$$

Please find the median of these m pair sums. Note that the median of a vector is the middle number after the vector is sorted. (Here we suppose the given n can always make m an odd number.)

More details:

- Input format:
  - Line 1 is an integer id $(1 \leq id \leq 6)$ indicating the subtask ID (to be defined later).
  - Line 2 is an integer n indicating the length of the vector $\mathbf{a}$.
  - Line 3 lists the vector's n elements separated by a space.
- Output format:
  - The median
- Ranges of variables:
  - $n \leq 2 \times 10^6$.
  - $0 \leq a_i \leq 10^9, \forall i$.
  - Others to be defined for each subtask explained below.
- Example test case:
  - Input

```
2
6
8 8 0 3 0 1
```

  - Output

```
8
```

  - Explanation of the above test case
    - 2 in line 1 of the input indicates the test set satisfies test group 2, where $n \leq 1000$
    - 6 in line 2 of the input indicates the vector has 6 elements, as listed in line 3.
    - Since the sums of all these 15 pairs are $0, 1, 1, 3, 3, 4, 8, 8, 8, 8, 9, 9, 11, 11, 16$, their median is 8.
- Subtask descriptions: To facilitate partial credit, we use subtask ID to indicate subtasks of different degrees of difficulty, as follows.
  - $id = 1$ (8 pts): $n \leq 2 \times 10^5$, and all elements in the vector has only a single value.

        - $id = 2$ (8 pts): $n \leq 10^3$.
        - $id = 3$ (8 pts): $n \leq 2 \times 10^5$, and all elements in the vector has only two distinct values.
        - $id = 4$ (8 pts): $n \leq 2 \times 10^5$, and $0 \leq a_i \leq 10^3, \forall i$.
        - $id = 5$ (4 pts): $n \leq 2 \times 10^5$.
        - $id = 6$ (4 pts): No restrictions.
    - Hints
        - Some subtasks are easier than the others. You can take advantage of this and get partial credits accordingly.
        - Be careful of overflow of data type "int".
        - If you want to tackle subtask with $id = 5$ and $id = 6$, here are more hints:
            - Can you design an efficient algorithm (with time complexity better than $O(n^2)$) to find the number of pairs whose sums are smaller than a given integer y? (In other words, for a given element, you can form pairs based on this element, and then find the number of pairs whose sums are smaller than y.)
            - If we use $count(\mathbf{b} < y)$ to represent the number of elements in $\mathbf{b}$ which is smaller than y, then we have the following observations:
                - If $count(\mathbf{b} < y) > m/2$, then y is larger than the median.
                - If $count(\mathbf{b} < y) < m/2$, then y is smaller than the median.

3. (30 pts) **Smallest K elements in a vector**: You are given a vector of N 32-bit signed integers. Find the first K smallest elements, and output them from the biggest to the smallest.

    More details:

    - Input format:
        - Line 1 is an integer T indicating the number of test cases.
        - T test cases follow, where each test case is composed of two lines.
            - The first line of the test case contains two positive integers, N and K, with K <= N.
            - The second line of the test case contains N integers of the vector.
    - Output format:
        - For every test case, output one line with K integers, that is, the first K smallest elements of the vector ordered from the biggest to the smallest.
    - Ranges of variables:

      $K \leq N \leq 10^6$.
    - Example test case:
        - Sample input

```
2
5 3
-3 -1 0 7 9
10 3
32 89 0 12 -11 32 -5 -11 91 7
```

        - Sample output:

```
0 -1 -3
-5 -11 -11
```

- Subtask descriptions
  1. $T \leq 100, N \leq 10^3, K \leq 10^2$ (6 pts)
  2. $T \leq 50, N \leq 10^5, K \leq 10^4$ (3 pts)
  3. $T \leq 5, K = 1$ (3 pts)
  4. $T \leq 5, K = 2$ (3 pts)
  5. $T \leq 5, K \leq 10^4$ (6 pts)
  6. $T \leq 2$ (9 pts)
- Hints:
  - Due to memory limit, it is impossible to store all N (10^6) integers in Subtask 3~6..

(Total score = 100)
